## PROPAGATION OF SURFACE ACOUSTIC WAVES ALONG THE FREE BOUNDARY OF A SATURATED POROUS MEDIUM

A. A. Gubaidullin and O. Yu. Boldyreva

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Frequency dependences of the velocity and attenuation coefficients of the waves propagating along a flat interface between a saturated porous medium and gas (vacuum) are studied. It is shown that the propagation of one or two surface modes is possible, depending on the parameters of the saturated porous medium and the conditions on the interface.

Key words: porous medium, boundary, acoustic waves.

**Introduction.** Surface acoustic waves (SAWs) are an important object of research in seismology and seismic prospecting. These are the main type of waves observed in earthquakes and explosions since, in propagating over a surface, they attenuate more slowly than body waves. In engineering, SAWs can be used for diagnostic and control of the quality of the surface and subsurface layers of samples.

In elastic solids, several types of surface waves with vertical polarization can exist [1, 2]. Any surface wave can be represented as a combination of inhomogeneous body waves in interfacing media for certain relations between the amplitudes of these waves (in this case, by an inhomogeneous wave is meant a two-dimensional monochromatic wave which propagates as a sinusoidal wave along the interface and has an exponential structure at a distance from the interface). Along the interface, all body components have the same propagation velocity (surface-wave velocity). At the same time, with distance from the interface, each component decreases or increases. If even one of the surface-wave components increases with depth, such a surface wave is called a leaky wave [1].

The main types of waves propagating along the free surface of a homogeneous elastic half-space or near its interface with another medium are Rayleigh waves, Stoneley waves, and Rayleigh pseudowaves.

Rayleigh wave propagate along the free surface of an elastic half-space and decay with distance from the surface into the depth of the medium. The velocity of these waves is slightly lower than the velocity of shear waves.

Stoneley waves can propagate along the interface between two solid half-spaces (for certain relations between the parameters of the interfacing media) or along the interface between a solid half-space and a fluid half-space (for any relations between the parameters of the solid and fluid media). The velocity of Stoneley wave is lower than the velocities of body waves in both media. As in Rayleigh waves, the disturbances in Stoneley waves decay with distance from the interface. Along the surface, Rayleigh and Stoneley waves propagate without dispersion and attenuation.

The Rayleigh pseudowave [1–4] can propagate along the interface between a solid half-space and a fluid half-space if the sound velocity in the fluid is lower than the Rayleigh wave velocity on the interface between the solid half-space and vacuum. The Rayleigh pseudowave velocity is higher than the sound velocity in the fluid but lower than the shear-wave velocity in the solid. Such a surface wave is a combination of inhomogeneous longitudinal and shear waves in the solid medium and an inhomogeneous wave in the fluid, and the amplitude of the latter increases exponentially with distance from the interface. In other words, the Rayleigh pseudowave is a leaky wave. The fact that an inhomogeneous wave in a fluid increases with distance from the interface may seem at first glance

Tyumen' Department of the Khristianovich Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, 625000 Tyumen'; gubai@tmn.ru; timms@ikz.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 50, No. 5, pp. 46–55, September–October, 2009. Original article submitted July 21, 2008.

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contradictory to the general laws of physics. In fact, the propagation of such a wave is accompanied by continuous reradiation of energy from the solid half-space into the fluid half-space [1]. In propagating, the Rayleigh pseudowave decreases in amplitude, i.e., the wave gradually decays due to reradiation and exists only in a certain region near the source.

Ultrasonic surface waves, including leaky waves propagating along the interface between a solid and a fluid, have also been observed experimentally [5]. The obtained velocity and attenuation coefficients of the Rayleigh pseudowave are in good agreement with theoretical values. Viktorov et al. note [5] that the presence of fluid on the interface leads to an increase in the phase velocity of surface Rayleigh waves.

In a porous medium, unlike in an elastic solid, longitudinal waves of two types can exist, as was predicted by Frenkel [6] and Biot [7] using a two-phase model of a porous medium and was later confirmed in experiments by Plona [8]. Therefore, the surface waves propagating in such media may have features not observed for surface waves in single-phase elastic media. Deresiewicz [9] was the first to study the propagation of surface waves in a porous medium using the Biot model [7]. The conditions on the interface between a porous medium with vacuum or a fluid were investigated by Deresiewicz and Skalak [10]. In studies of wave processes near the interface between a porous medium and a fluid or vacuum, one usually considers the case of completely open pores with free fluid flow through the interface, the case of completely closed pores with no fluid flow through the interface, and the intermediate cases of limited fluid flow through the interface.

Using Biot theory in a high-frequency approximation, Feng and Johnson [11] performed a detailed study of surface waves propagating along the interface between a porous solid and a fluid. They carried out calculations over a wide range of the elastic parameters of a the porous skeleton in the presence of open and closed pores on the boundary of the porous solid. It has been found that, in the high-frequency range, one, two or three surface modes can propagate, depending on the elastic parameters of the skeleton and fluid and the conditions on the interface. Among the solutions obtained for surface waves, Feng and Johnson [11] distinguish a true mode, whose velocity is lower than the velocities of all body waves in both interfacing media, and Rayleigh and Stoneley pseudomodes. The velocity of the Stoneley pseudomode is higher than the velocity of a slow longitudinal wave in the porous medium, but lower than sound velocity in the fluid adjacent to the porous medium. The velocity of the Rayleigh pseudomode is somewhat lower than the shear-wave velocity in the fluid. Each pseudomode is leaky since the amplitude of one or two of its body constituents increases as the wave recedes from the interface between the media. The true mode propagates along the interface without decay, and the pseudomodes propagating along the boundary surface decay due to energy reradiation into the depth of the porous medium and the fluid (or only the porous medium).

Feng and Johnson [12] obtained an expression for Green function which allows one to determine the response of the porous medium-fluid system to the pressure pulses generated by a source in the fluid. It has been established that a pulse source located in the fluid near the interface with the porous medium generates surface modes in this medium that correspond to the surface modes obtained in [11]. Although the results of [12] were obtained using Green function without invoking the assumptions of the existence of surface modes on the porous medium-fluid interface, they are in good agreement with the results of [11].

The experiments of Nagy [13] demonstrated a new surface mode (different from surface waves in solids) generated on the surface of a fluid-saturated porous sample, whose velocity agreed with the theoretical value for the true surface mode predicted in [11]. Adler and Nagy [14] describe the generation and detection of surface waves on the boundary of natural rocks and synthetic porous materials with a wavy (periodic) surface which are partially or completely immersed in a fluid. The velocity of the surface mode observed on the interface between a fluid-saturated porous sample and air was approximately 60% of the shear-wave velocity, which is close to the theoretical value for the true surface wave [11].

Edel'man [15] performed a theoretical study of the waves propagating along the interface between a porous medium and vacuum, fluid or another porous medium. Asymptotic expressions for surface-waves velocities were obtained in the high-frequency approximation. The existence of roots of the dispersion equations corresponding to leaky surface waves was proved.

Gubaidullin and Boldyreva [16, 17] studied for the first time the frequency dependences of the velocity and attenuation coefficients of the surface waves propagating along the interface between a saturated porous medium and

a fluid. It has been shown that, depending on the parameters of the saturated porous medium and the conditions on the interface, the propagation of one, two or three surface modes is possible, each of which is a true mode or a pseudomode. The results are in good agreement with the well-known results obtained for the high-frequency approximation.

The purpose of the present work is to study the surface waves propagating along the interface between a fluid-saturated porous medium and a gas over a wide frequency range and the frequency dependences of the velocity and attenuation coefficients of the obtained surface modes.

**Equations of Motion.** In the study we use a two-phase model of a porous medium with an elastic skeleton [18, 19]. The linearized equations of motion for a saturated porous medium are written as

$$\begin{aligned} \frac{\partial \rho_s}{\partial t} + \rho_{s0} \nabla^k v_s^k &= 0, \qquad \frac{\partial \rho_f}{\partial t} + \rho_{f0} \nabla^k v_f^k = 0, \\ \rho_{s0} \frac{\partial v_s^k}{\partial t} &= -\alpha_{s0} \nabla^k p_f + \nabla^l \sigma_{2*}^{kl} + F^k, \qquad \rho_{f0} \frac{\partial v_f^k}{\partial t} = -\alpha_{f0} \nabla^k p_f - F^k, \\ (\sigma_{s*}^{kn})' &= \alpha_{s0} (\lambda_* \delta^{kn} \varepsilon_s^{mm} + 2\mu_* \varepsilon_s^{kn} + \nu_* \delta^{kn} p_f'), \qquad \nu_* = (\lambda_* + 2\mu_*/3)/K_s, \\ \frac{\partial \varepsilon_s^{kn}}{\partial t} &= \frac{1}{2} (\nabla^k v_s^n + \nabla^n v_s^k), \\ \rho_f' &= \alpha_f' \rho_{f0}^0 + \alpha_{f0} \rho_f^{0\prime}, \qquad \rho_s' = \alpha_s' \rho_{s0}^0 + \alpha_{s0} \rho_s^{0\prime}, \qquad \alpha_f' + \alpha_s' = 0, \\ p_s' &= K_s \rho_s^{0\prime} / \rho_{s0}^0, \qquad p_f' = K_f \rho_f^{0\prime} / \rho_{f0}^0, \\ p_{s*}' &= \alpha_{s0} (p_s' - p_f') + \alpha_s' (p_{s0} - p_{f0}), \qquad p_{s*} = -\sigma_{s*}^{mm}/3. \end{aligned}$$

Here the superscripts correspond to space coordinates (the summation is performed over repeated indices); the subscripts s and f correspond to the solid or fluid phases, respectively; the lower subscript 0 denotes the unperturbed values of a quantity, and the prime denotes a deviation of a quantity from its unperturbed value ( $w' = w - w_0$ );  $\alpha_j$ ,  $\rho_j^0$ ,  $\alpha_j^0$ ,  $\alpha_j^0$ , and  $v_j$  are the bulk concentration, the reduced and true densities, and velocity of the *j*th phase (j = s or j = f), respectively,  $p_f$  is the pressure in the fluid,  $\sigma_{s*}$  is the reduced stress in the skeleton of the medium, F is the interfacial force,  $\alpha_s \lambda_*$  and  $\alpha_s \mu_*$  are the elastic moduli of the skeleton of the porous medium,  $\varepsilon_s$  are the strains of the solid phase,  $p_s$  is the pressure in the solid phase, and  $K_s$  and  $K_f$  are the bulk elastic moduli of the solid-phase and fluid, respectively.

In the case of propagation of a monochromatic wave at a frequency  $\omega$ , the interphase force F is expressed as [19]

$$F = F_m + F_\mu + F_B, \qquad F_m = 0.5\eta_m \alpha_{f0} \alpha_{s0} \rho_{f0}^0 i\omega (v_f - v_s),$$
$$F_\mu = \eta_\mu a_*^{-2} \alpha_{f0} \alpha_{s0} \mu_f (v_f - v_s), \qquad F_B = \eta_B a_*^{-1} \alpha_{f0} \alpha_{s0} \sqrt{2\rho_{f0}^0 \mu_f \omega} \ (1+i)(v_f - v_s).$$

Here  $F_m$  is the added-mass force due to the inertial interaction between the phases,  $F_{\mu}$  is the Stokes friction force,  $F_{\rm B}$  is an analog of the Basset force due to the nonstationarity of the viscous interface layer near the interface between the fluid and the solid phase, *i* is the imaginary unit,  $\mu_f$  is the dynamic viscosity of the fluid,  $a_*$  is the characteristic size of pores or grains, and  $\eta_m$ ,  $\eta_\mu$ , and  $\eta_{\rm B}$  are the coefficients of inertial, viscous, and viscous-inertial interactions between the phases which depend on the structure of the medium.

The model of a two-phase porous medium with an elastic skeleton used in the present work is equivalent to the model [7] employed by foreign researchers in studying processes in porous media. The main difference is in the form of the expression for the interfacial viscous force  $F_{\mu} + F_{\rm B}$ . In the foreign literature, this force, which asymptotically coincides with  $F_{\mu}$  or  $F_{\rm B}$  in the transition to low or high frequencies, respectively, is usually written using one expression for all frequencies [20, 21]. In the model used in the present work, the interfacial viscous force is written as the sum of quasistationary and nonstationary components, with the force  $F_{\mu}$  dominating at low frequencies and the force  $F_{\rm B}$  at high frequencies. **Surface Waves.** Let a fluid-saturated porous medium occupy a half-space z > 0 and be adjacent to a gas region z < 0. In the half-space z < 0, the gas motion is described by the linearized equations

$$\frac{\partial \tilde{\rho}_g}{\partial t} + \tilde{\rho}_{g0} \nabla^n \tilde{v}_g^n = 0, \qquad \tilde{\rho}_{g0} \frac{\partial \tilde{v}_g}{\partial t} + \nabla \tilde{p}_g = 0, \qquad \tilde{p}_g' = \frac{K_g \tilde{\rho}_g'}{\tilde{\rho}_{g0}}.$$

We consider two-dimensional motion that corresponds to the propagation of a surface wave along the x axis, assuming that all quantities depend only on x and z. Harmonic surface waves can be treated as combinations of inhomogeneous body waves in the porous medium and gas. The displacement potentials for a porous medium and gas are written as

$$\boldsymbol{v}_{j} = \frac{\partial \boldsymbol{u}_{j}}{\partial t}, \qquad \boldsymbol{u}_{j} = \operatorname{grad} \Phi_{j1} + \operatorname{grad} \Phi_{j2} + \operatorname{rot} \boldsymbol{\Psi}_{j}, \qquad \boldsymbol{\Psi}_{j} = \{0, \Psi_{j}, 0\},$$
$$\tilde{\boldsymbol{v}}_{g} = \frac{\partial \tilde{\boldsymbol{u}}_{g}}{\partial t}, \qquad \tilde{\boldsymbol{u}}_{g} = \operatorname{grad} \tilde{\Phi}_{g},$$
$$\begin{pmatrix} \Phi_{j1} \\ \Phi_{j2} \\ \Psi_{j} \\ \tilde{\Phi}_{g} \end{pmatrix} = \begin{pmatrix} A_{j1} e^{-\gamma_{1}z} \\ A_{j2} e^{-\gamma_{2}z} \\ iB_{j} e^{-\beta z} \\ \tilde{A}_{g} e^{\gamma_{2}z} \end{pmatrix} \exp i(\omega t - kx).$$

Here  $\Phi_{j1}$ ,  $\Phi_{j2}$ , and  $\Psi_j$  are the scalar and vector potentials for the solid  $(j \equiv s)$  and fluid  $(j \equiv f)$  phases of the porous medium, and  $\tilde{\Phi}_g$  is the scalar potential for the gas adjacent to the porous medium. It should be noted that the wavenumbers of the surface wave (k), longitudinal and shear waves in the porous medium  $(k_{l1}, k_{l2}, \text{ and } k_t)$ , sound waves in the gas  $(\tilde{k}_g)$ , and the coefficients  $\gamma_1$ ,  $\gamma_2$ ,  $\beta$ , and  $\gamma_g$  are linked by the relations

$$k_{l1}^2 = k^2 - \gamma_1^2, \qquad k_{l2}^2 = k^2 - \gamma_2^2, \qquad k_t^2 = k^2 - \beta^2, \qquad \tilde{k}_g^2 = k^2 - \gamma_g^2$$

On the interface z = 0, we specify continuity conditions for the normal components of the total stress  $\sigma$  in the medium ( $\sigma^{ik} = -\delta^{ik}p_f + \sigma^{ik}_{s*}$ ), equality of the volumes of the fluids flowing through the interface, and a linear relation between the pressure jump and the normal velocity of the fluid in the porous medium [10, 11]:

$$z = 0: \qquad \sigma_{s*}^{xz} = 0, \quad -\sigma_{s*}^{zz} + p_f = \tilde{p}_g, \quad p_f - \tilde{p}_g = T\alpha_f (u_f^z - u_s^z), \quad \alpha_f (u_f^z - u_s^z) = \tilde{u}_g^z - u_s^z.$$

The parameter T characterizes the boundary surface of the porous medium: the value T = 0 corresponds to the case of open pores with free fluid flow through the boundary of the porous medium,  $T = \infty$  to the case of completely closed pores on the boundary, and the intermediate finite values T > 0 to limited flow through the interface.

The interface between the porous medium and vacuum is free; therefore, on the boundary surface, the following conditions should be satisfied:

$$z = 0; \qquad \sigma_{s*}^{xz} = 0, \quad -\sigma_{s*}^{zz} + p_f = 0, \quad p_f = T\alpha_f (u_f^z - u_s^z).$$

Using a standard procedure [1, 9, 11], we obtain a dispersion relation which allows us to determine the wavenumber k and the coefficients  $\gamma_1 = \sqrt{k^2 - k_{l1}^2}$ ,  $\gamma_2 = \sqrt{k^2 - k_{l2}^2}$ ,  $\beta = \sqrt{k^2 - k_t^2}$ , and  $\gamma_g = \sqrt{k^2 - \tilde{k}_g^2}$  as functions of the frequency  $\omega$ . The complex roots k of the dispersion equation are obtained using a numerical algorithm for searching and refining the roots of the equations in the complex region. Next, the phase velocity and the spatial attenuation coefficient ( $C = \omega / \operatorname{Re} k$  and  $\delta = -\operatorname{Im} k$ ) are calculated.

**Calculation Results.** The calculations were performed over a wide range of frequencies and parameters of the porous medium for the cases of open or closed pores on the interface between the porous medium and gas. For comparison, calculations were performed for the surface waves propagating along the free boundary (with vacuum) of the saturated porous medium.

The calculations results show that, depending on whether the boundary pores are open or closed, one or two surface modes can propagate: a Rayleigh pseudomode in the case of open pores and a slow surface mode and a Rayleigh pseudomode in the case of closed pores. The velocity of the Rayleigh pseudomode is close to the shear (shear) wave velocity in the saturated porous medium. If the porous medium is adjacent to vacuum, the slow surface mode is a true one because its velocity is lower than velocities of all body [fast (strain) and slow (filtration) longitudinal and shear] waves in this medium. If the saturated porous medium is adjacent to a gas, the slow surface

Interphase	Type of surface wave		
	Open pores	Closed pores	
Fluid saturated porous medium–gas (air)	Leaky Rayleigh pseudomode $(C_g < C < C_t;$ Re $\gamma_g < 0$ at all frequencies and Re $\gamma_2 < 0$ at high frequencies)	Leaky Rayleigh pseudomode $(C_g < C < C_t;$ Re $\gamma_g < 0$ at all frequencies and Re $\gamma_2 < 0$ at high frequencies)	
		Leaky true mode $(C < C_{l2} \text{ at all frequencies},$ $C > C_g \text{ at high frequencies, and}$ $\operatorname{Re} \gamma_g < 0$ at all frequencies)	
Fluid saturated porous medium–vacuum	Rayleigh pseudomode $(C_f < C < C_t)$ , which is leaky at high frequencies $(\operatorname{Re} \gamma_2 < 0)$	Rayleigh pseudomode $(C_f < C < C_t)$ , which is leaky at high frequencies $(\operatorname{Re} \gamma_2 < 0)$	
		True mode $(C < C_{l2})$	

Characteristics of Surface Waves	
Propagating on the Interface between Fluid-Saturated Porous Medium and Gas or	<sup>·</sup> Vacuum

mode is not true in the strict sense since its velocity is lower than the velocities of all body waves in the porous medium, but at high frequencies, it exceeds the sound velocity in the gas adjacent to the porous medium. The velocity and attenuation coefficients of the slow surface mode are close to those of the true surface mode propagating along the free surface of the saturated porous medium; therefore, it can be called a true mode. The coincidence (with accuracy up to the thickness of the line in the plot) of the phase velocities and spatial attenuation coefficients of the surface waves on the interface between the porous and gas or vacuum (on the free surface of the porous medium) is due to the fact that the wave resistance of the gas medium is negligibly small compared to the wave resistance of the saturated porous medium. Therefore, in the calculations, we can confine ourselves to the simpler case of the free surface of the porous medium.

The obtained results are given in Table 1, which lists the surface modes propagating on the interface between the fluid-saturated porous medium and gas or vacuum for the cases of open or closed pores on the interface and their properties. The velocity range of the surface mode is given and the frequencies for which the pseudomode is leaky or true are indicated.

In the calculations, it was assumed that the porous medium is water-saturated quartz sand with parameters  $\alpha_f = 0.33$ ,  $a_* = 0.25$  mm,  $\lambda_* = \mu_* = 8 \cdot 10^9$  Pa,  $\eta_m = 1$ ,  $\eta_\mu = 100$ , and  $\eta_B = 1.5$ . The gas was considered to be air.

Figure 1 shows the phase velocity and spatial attenuation coefficient of the surface modes in the case of closed pores on the interface. For comparison, the same figure shows the velocities and attenuation coefficients of body waves in the porous medium. Figure 2 shows frequency dependences of the dimensionless coefficients  $d_{l1} = \text{Re}(\gamma_1)\lambda$ ,  $d_{l2} = \text{Re}(\gamma_2)\lambda$ ,  $d_t = \text{Re}(\beta)\lambda$ , and  $d_g = \text{Re}(\gamma_g)\lambda$  ( $\lambda$  is the wavelength) which characterize the exponential decrease or increase in the body components of the surface modes with distance from the interface.

The Rayleigh pseudomode is leaky because over the entire frequency range,  $d_g < 0$ , and, hence, at a distance from the interface, the amplitude of its g components increases due to energy reradiation into the half-space filled with the gas. At high frequencies, the amplitude of the l2 components of the Rayleigh pseudomode also increases; however, as the frequency decreases, the increase in the l2 component due to reradiation is replaced by its decrease due to a reduction in the viscous interfacial interaction.

Frequency dependences of the velocity of the Rayleigh pseudomode (propagating along the interface between the porous medium and gas) and the coefficients characterizing the decrease or increase in its body components for the case of open pores are presented in Figs. 3 and 4. In the case of open pores on the interface, the properties of the Rayleigh pseudomode are close to the properties of the Rayleigh pseudomode in case of closed pores on the interface.

**Conclusions.** Thus, the frequency dependences of the velocity and attenuation coefficients of the surface waves propagating along the interface between a fluid-saturated porous medium and a gas or vacuum were studied.



Fig. 1. Frequency dependences of the phase velocity C (a) and the spatial attenuation coefficient  $\delta$  (b) of the body longitudinal waves (curves 1 and 2) and shear wave (curve 3) in the saturated porous medium, and the true mode (curve 4) and Rayleigh pseudomode (curve 5) in the case of closed pores on the interface between water-saturated quartz sand and air.



Fig. 2. Frequency dependences of the attenuation coefficients of the body components of the true mode (a) and Rayleigh pseudomode (b) at a distance from the interface between water-saturated quartz sand and air equal to the wavelength in the case of closed pores on the interface.



Fig. 3. Frequency dependences of the phase velocity C (a) and the spatial attenuation coefficient  $\delta$  (b) of the body longitudinal waves (curves 1 and 2) and shear wave (curve 3) in the saturated porous medium and the Rayleigh pseudomode (curve 4) in the case of open pores on the interface between water-saturated quartz sand and air.



Fig. 4. Frequency dependences of the attenuation coefficients of the body components of the Rayleigh pseudomode at a distance from the interface between water-saturated quartz sand and air equal to the wavelength in the case of open pores on the interface.

The results shows that, depending on the parameters of the saturated porous medium and the conditions on the interface, one or two surface modes can propagate — a Rayleigh pseudomode and a true mode, each of which is a leaky wave which decays due to both interfacial force interaction in the porous medium and reradiation into the inhomogeneous wave in the half-space filled with the gas. The velocity of the true surface mode is close to the velocity of a slow longitudinal (filtration) wave in the porous medium. The velocity of the Rayleigh pseudomode is

somewhat lower than the shear-wave velocity in the porous medium, and the properties of this pseudomode depend weakly on whether the boundary pores are open or closed. In the case of vacuum, the second mode is true.

In the case of a fluid-saturated porous medium adjacent to a gas or vacuum, the phase velocity and the spatial attenuation coefficient of surface waves were shown to be nearly identical. This is due to the fact the wave resistance of the gas is considerably lower than the wave resistance of the saturated porous medium.

In the case of vacuum, the Rayleigh pseudomode is leaky only at high frequencies. Its component corresponding to the filtration wave increases at high frequencies due to reradiation and decreases at low frequencies due to viscous interfacial interaction.

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